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Abstract. This work was concerned with research on fundamental mechanics and mathematics of *large deformation induced failures* in nonlinear solids. The specific area investigated was that of *void nucleation and growth* due to large deformations in nonlinear solids. Research on cavitation phenomena, which serve as a precursor to *fracture*, is crucial to the understanding of *failure mechanisms* in rubber-like solids (e.g. polymers, solid rocket propellants) and of ductile fracture processes in metals. Mathematically, the work involves investigation of singular solutions of the second-order quasilinear system of partial differential equations describing equilibrium states of nonlinearly elastic bodies. For radially symmetric deformations, the basic problem reduces to a bifurcation problem for a single second-order nonlinear ordinary differential equation. Particular emphasis was placed on the effect of *material inhomogeneity, compressibility and anisotropy* on void nucleation and growth.

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ANNUAL TECHNICAL REPORT

AFOSR Grant F49620-92-J-0112
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Title: Large Deformation Failure Mechanisms in Nonlinear Solids

Submitted to: Air Force Office of Scientific Research
110 Duncan Ave., Suite B115
Bolling Air Force Base
Washington, DC 20332-0001

Attention: Dr. Arje Nachman
Program Director
Applied Analysis

Author: Cornelius O. Horgan, Professor
Department of Applied Mathematics
School of Engineering and Applied Science
University of Virginia
Charlottesville, VA 22901
(804) 924-7230

1. Summary of Research Results:

This work is concerned with research on the fundamental mechanics and mathematics of large deformation induced failures in nonlinear solids. The specific area investigated was that of void nucleation and growth due to large deformations in nonlinear solids. Research on cavitation phenomena, which serve as a precursor to fracture, is crucial to the understanding of failure mechanisms in rubber-like solids (e.g. polymers, solid rocket propellants) and of ductile fracture processes in metals. Mathematically, the work involved investigation of singular solutions of the second-order quasilinear system of partial differential equations describing equilibrium states of nonlinearly elastic bodies. For radially symmetric deformations, the basic problem reduces to a bifurcation problem for a single second-order nonlinear ordinary differential equation. Particular emphasis was placed on the effect of material inhomogeneity, compressibility and anisotropy on void nucleation and growth. Studies on the micromechanics of particles, in particular the correlation of inclusions to void formation, are receiving much attention from the solid mechanics, applied mathematics and materials science communities. This research emphasis has been highlighted in a recent National Committee report on Solid Mechanics Research Directions. The work also has impact on failure mechanisms due to large deformations in anisotropic and composite materials. Compared to the vast amount of information available on small deformations of such materials, results on large deformations remain virtually unexplored. Considerations of large deformations in anisotropic or composite materials often lead to striking differences from predictions of corresponding linearized theories. In view of the rapid utilization of advanced composite materials in present day Air Force technology, studies on the fundamental mechanics and mathematics of large deformations of such materials promise to have widespread impact on the AFOSR mission.

In what follows, we summarize briefly the main results obtained in the list of publications [1-8] attached. Detailed abstracts of each paper are given with the list of publications.

Cavity nucleation involving the formation and subsequent growth and coalescence of voids from microscopic inclusions leading to ductile fracture in metals and alloys has long been of concern to metallurgists. The phenomenon of sudden void formation ("cavitation") also arises in rubber elasticity and is also of interest in soil mechanics in connection with pile driving. Because of the role such cavitation phenomena play in failure processes for metals and polymers, such problems have attracted much recent attention from the solid mechanics and applied mathematics communities.

The impetus for much of the recent theoretical developments has been supplied by the mathematical work of Ball in 1982 concerned with *singular* solutions in nonlinear elasticity.

Ball has studied a class of *bifurcation problems* for the equations of nonlinear elasticity which model the appearance of a cavity in the interior of an apparently solid homogeneous isotropic elastic body once a critical load has been attained. The alternative interpretation for such problems in terms of the growth of a *pre-existing* micro-void is more attractive from a physical point of view. It is important for us to emphasize here that the idealized mathematical treatment using a bifurcation approach does correctly predict the critical load at which a *pre-existing* microvoid will undergo sudden rapid growth. The critical load is automatically given by this approach - imposition of a failure criterion is not necessary. The bifurcation approach to cavitation may thus be viewed as a convenient analytical tool which furnishes values of the critical load. A treatment of the pre-existing traction-free microvoid problem is more complicated analytically. It is also important to note that *infinitesimal* theories of isotropic solid mechanics (including the classical theories of small strain plasticity) do *not* predict this cavitation phenomenon.

In [1], the effect of material *anisotropy* was examined. Very little is known about large deformations of anisotropic nonlinearly elastic materials. In [1] we were able to obtain closed-form analytic solutions to a cavitation problem for a transversely-isotropic incompressible nonlinearly elastic sphere. One of the striking results found was that the bifurcation could be either to the right (supercritical) or to the left (subcritical) *depending on the degree of anisotropy* of the material. In the latter case, the cavity has finite radius on first appearance. This discontinuous change in stable equilibrium configurations ("snap cavitation") is reminiscent of the snap-through buckling phenomenon observed in certain structural mechanics problems. A similar snap cavitation phenomenon has been encountered recently by Antman and Negron-Marrero in the study of radially symmetric equilibrium states of *homogeneous anisotropic compressible* nonlinearly elastic bodies, and by Horgan and Pence for *composite* incompressible *isotropic* materials. Such a striking material instability does *not* occur for homogeneous isotropic materials. The radial anisotropy considered in [1] arises in material processing, for example, due to the thermal gradients arising in metal casting. Our results show that such anisotropy can lead to material failure.

The effects of material anisotropy and inhomogeneity on void nucleation and growth in incompressible anisotropic nonlinearly elastic solids are examined in [2]. A bifurcation problem is considered for a *composite* sphere composed of two arbitrary homogeneous incompressible nonlinearly elastic materials which are transversely isotropic about the radial direction, and perfectly bonded across a spherical interface. Under a uniform radial tensile dead-load, a branch of radially symmetric configurations involving a traction-free internal cavity bifurcates from the undeformed configuration at sufficiently large loads. Several types of bifurcation are found to occur. Explicit conditions determining the type of bifurcation are established for the general

transversely isotropic composite sphere. In particular, if each phase is described by an explicit material model which may be viewed as a generalization of the classic neo-Hookean model to anisotropic materials, phenomena which were *not* observed for the homogeneous anisotropic sphere [1] nor for the composite neo-Hookean sphere (Horgan and Pence, 1989) may occur. The stress distribution as well as the possible prevention of interface debonding due to cavitation are also examined for the general composite sphere.

The effects of *material compressibility* were investigated in [3]. The absence of the zero volume change constraint complicates the analytical solution of problems, so that until quite recently, the major advances in nonlinear elasticity theory were made for incompressible materials. In earlier work a scheme was developed for simplifying the analytical solution of axisymmetric elastostatic problems for nonlinearly elastic compressible materials. It was shown how the *second-order* nonlinear ordinary differential equation arising from equilibrium could be reduced to a pair of *first-order* equations. This led to new analytical solutions (in closed form) for a variety of compressible material models. Another aspect of compressible materials undergoing large deformations was investigated in [3] following on earlier results published in 1992. The basic issue examined is to identify the class of compressible materials for which volume preserving (i.e. isochoric) anti-plane shear or azimuthal (or circular) shear deformations can occur in hollow circular tubes. (The anti-plane shear deformations are of concern in Mode III fracture of materials). It was shown in [3] for the azimuthal shear problem that a variety of compressible material models can sustain such volume preserving deformations. The results have implications for the design of experiments and for constitutive modeling.

In [4,5], we return to the issue of slow stress diffusion and Saint-Venant end effects in composite structures. Thin-walled structures such as aircraft and automotive parts, rocket casings, helicopter blades and containment vessels are often constructed of layers of anisotropic, filament or fiber-reinforced materials which must be designed to remain elastic. The extent to which *local* stresses, such as those produced by fasteners and at joints, can penetrate girders, beams, plates and shells must be understood by the designer. Thus a distinction must be made between *global* structural elements (where Strength of Materials or other approximate theories may be used) and *local* elements which require more detailed (and more costly) analyses based on exact elasticity. The neglect of end effects is usually justified by appeals to some form of Saint-Venant's principle and years of experience with *homogeneous isotropic elastic structures* has served to establish this standard procedure. Saint-Venant's principle also is the fundamental basis for static mechanical tests of material properties. Thus property measurements are made in a suitable *gage section* where *uniform* stress and strain states are induced and local effects due to clamping of the specimen are neglected on invoking Saint-Venant's principle. Such traditional

applications of Saint-Venant's principle require major modifications when strongly anisotropic and composite materials are of concern. For such materials, local stress effects persist over distances *far greater* than is typical for isotropic metals. In [4], we describe plane elastostatic problems where anisotropy induces such extended Saint-Venant end zones. The paper is a review and a comprehensive list of references is given to original work where details of the analyses may be found. A more comprehensive review, which included three-dimensional problems, effects of nonlinearity, and dynamic effects, is provided in [5]. The implications of such extended end zones due to anisotropy are far reaching in the proper analysis and design of structures using advanced composite materials.

Explicit analytic results of the type described above are crucial to the complete analysis of *local* or *end effects* in anisotropic or composite materials and structures. Previous work has shown that such end effects decay *much more slowly* than in isotropic materials. For transversely-isotropic (or specially orthotropic) materials, our earlier work has led to *specific design formulas* for the distance beyond which Saint-Venant edge effects can be neglected. The results, which have important implications for the experimental techniques used to measure material properties, have led to modifications of the ASTM standard test and are now quoted routinely in text- and hand-books on mechanics of composite materials. Our current research deals with more complicated degrees of anisotropy, including the general orthotropic symmetry relevant to the off-axis tension test, and with effects of nonlinearity. Analytic results of the type obtained are *crucial complements to large-scale computational analyses.*

In [6], plane deformations of a rectangular strip, composed of an homogeneous fully anisotropic linearly elastic material, are considered. The strip is in equilibrium under the action of end loads, with the lateral sides traction-free. Two conservation properties for certain cross-sectional stress measures are established, generalizing previously known results for the isotropic case. It is noteworthy that in the first of these conservation laws only one of the off-axis elastic constants appears explicitly while in the second only the opposite off-axis constant appears explicitly. Such conservation properties are useful in assessing the influence of material anisotropy on Saint-Venant's principle, as well as in establishing convexity properties for cross-sectional stress measures. In particular, it is anticipated that the results should be useful in determining the extent of edge effects in the off-axis testing of anisotropic and composite materials.

In the context of linear elasticity theory, Saint-Venant's Principle is often used to justify the neglect of edge effects when determining stresses in a material. This is valid in the case of an isotropic material. However for the more general anisotropic material, experimental results have shown that edge effects may persist much farther into the material than in the isotropic case and cannot be neglected. In [7] the effect of material anisotropy on the exponential decay rate for

stresses in a semi-infinite elastic strip is examined. A linear elastic semi-infinite strip in a state of plane stress/strain subject to a self-equilibrated end load is considered first for an orthotropic material and then for the most general anisotropic material. The problem is governed by a fourth-order elliptic partial differential equation with constant coefficients. Conservation properties of the solution are derived to help in determining decay rate estimates. Energy methods are then used to provide lower bounds on the actual decay rate. Both analytic and numerical estimates are obtained in terms of the elastic constants of the material and results are shown for a set of specific materials. When compared with the exact decay rate computed numerically from the eigenvalues of a fourth-order ordinary differential equation, the results in some cases show a high degree of accuracy not achieved previously. Results of the type obtained here have several practical applications, for example, in the mechanical testing of anisotropic and composite materials and in assessing the influence of fasteners, joints, etc. in composite structures.

Anti-plane shear deformations of a cylindrical body, with a single displacement field parallel to the generators of the cylinder and independent of the axial coordinate, are one of the simplest classes of deformations that solids can undergo. They may be viewed as complementary to the more familiar plane deformations. Anti-plane (or longitudinal) shear deformations have been the subject of considerable recent interest in *nonlinear* elasticity theory for homogeneous isotropic solids. In contrast, for the *linear* theory of *isotropic* elasticity, such deformations are usually not extensively discussed. The purpose of the paper [8] is to demonstrate that for *inhomogeneous anisotropic linearly elastic solids* the anti-plane shear problem *does* provide a particularly tractable and illuminating setting within which effects of anisotropy and inhomogeneity may be examined. We consider infinitesimal anti-plane shear deformations of an inhomogeneous anisotropic linearly elastic cylinder subject to prescribed surface tractions on its lateral boundary whose only nonzero component is axial and which does not vary in the axial direction. In the absence of body forces, *not all arbitrary anisotropic cylinders* will sustain an anti-plane shear deformation under such tractions. Necessary and sufficient conditions on the elastic moduli are obtained which *do allow* an anti-plane shear. The resulting boundary value problems governing the axial displacement are formulated. The most general elastic symmetry consistent with an anti-plane shear is described. There are at most 15 independent elastic coefficients associated with such a material. In general, there is a *normal* axial stress present, which can be written as a linear combination of the two dominant shear stresses. For a material with the cylindrical cross-section a plane of elastic symmetry (monoclinic, with 13 moduli) the normal stress is no longer present. For *homogeneous* materials, it is shown how the governing boundary-value problem can be transformed to an equivalent isotropic problem for a transformed cross-sectional domain. Applications to the issue of assessing the influence of anisotropy and inhomogeneity on

the decay of Saint-Venant end effects are described.

Student Participation:

Dr. Debra A. Polignone, former Ph.D. student, has been actively involved in this research. Her Ph.D. Dissertation, which resulted in publications [1, 2], was defended on May 3, 1993. Dr. Polignone was awarded a National Defense Science and Engineering Graduate Fellowship (NDSEGF) for 1990-1993, sponsored by the U.S. Air Force. Dr. Polignone was a post-doctoral Fellow at Center for Nonlinear Analysis, Carnegie-Mellon University, Pittsburgh, PA 1993-94. (Fellowship awarded as a result of national competition). She began a tenure-track Assistant Professorship at Dept. of Mathematics, University of Tennessee, Knoxville, June 1994.

A second student, Dr. Kristin L. Miller, defended her Ph.D. Dissertation in April 25, 1994. She was also awarded a NDSEGF, sponsored by U.S. Air Force, for 1991-1994. Her work resulted in publications [6, 8] and two forthcoming papers. She was awarded a NRC Postdoctoral fellowship in The Computational and Applied Mathematics Division, NIST for 1994-1996. She decided to accept a permanent position as Cryptologic Mathematician, National Security Agency (NSA), Fort Meade, MD, Sept. 1994. A third student, Sarah C. Baxter is now preparing her Ph.D. Dissertation. Her work is supported in part by grants from the Virginia Space Grant Consortium and ARO.

Publications

(List of recent papers, theses and dissertations that have been supported, or partially supported, by AFOSR.)

1. Polignone, Debra A., and Horgan, C. O., "Cavitation for Anisotropic Incompressible Non-linearly Elastic Spheres," *J. of Elasticity* **33**, 1993, 27-65.

Abstract. In this paper, the effect of *material anisotropy* on void nucleation and growth in *incompressible* nonlinearly elastic solids is examined. A bifurcation problem is considered for a solid sphere composed of an incompressible homogeneous nonlinearly elastic material which is transversely isotropic about the radial direction. Under a uniform radial tensile dead-load, a branch of radially symmetric configurations involving a traction-free internal cavity bifurcates from the undeformed configuration at sufficiently large loads. Closed form analytic solutions are obtained for a specific material model, which may be viewed as a generalization of the classic neo-Hookean model to anisotropic materials. In contrast to the situation for a neo-Hookean sphere, bifurcation here may occur locally either to the right (supercritical) or to the left (subcritical), depending on the degree of anisotropy. In the latter case, the cavity has finite radius on first appearance. Such a discontinuous change in stable equilibrium configurations is reminiscent of the snap-through buckling phenomenon of structural mechanics. Such dramatic cavitational instabilities were previously encountered by Antman and Negron-Marrero (1987) for anisotropic *compressible* solids and by Horgan and Pence (1989) for *composite* incompressible spheres.

2. Polignone, Debra A., and Horgan, C. O., "Effects of material anisotropy and inhomogeneity on cavitation for composite incompressible anisotropic nonlinearly elastic spheres," *Int. J. of Solids & Structures* **30**, (1993), 3381-3416.

Abstract. The effects of *material anisotropy* and *inhomogeneity* on void nucleation and growth in incompressible anisotropic nonlinearly elastic solids are examined. A bifurcation problem is considered for a composite sphere composed of two arbitrary homogeneous incompressible nonlinearly elastic materials which are transversely isotropic about the radial direction, and perfectly bonded across a spherical interface. Under a uniform radial tensile dead-load, a branch of radially symmetric configurations involving a traction-free internal cavity bifurcates from the undeformed configuration at sufficiently large loads. Several types of bifurcation are found to occur. Explicit conditions determining the type of bifurcation are established for the general transversely isotropic composite sphere. Dramatic cavitational instabilities reminiscent of the snap-through buckling phenomena of structural mechanics are found to occur. In

particular, if each phase is described by an explicit material model which may be viewed as a generalization of the classic neo-Hookean model to anisotropic materials, phenomena which were not observed for the homogeneous anisotropic sphere [Antman and Negron-Marrero, 1987, Polignone and Horgan, 1993] nor for the composite neo-Hookean sphere [Horgan and Pence, 1989] may occur. The stress distribution as well as the possible role of cavitation in preventing interface debonding are also examined for the general composite sphere.

3. Polignone, Debra A., and Horgan, C. O., "Pure Azimuthal Shear of Compressible Non-linearly Elastic Circular Tubes," *Quarterly of Applied Mathematics,* **52**, 1994, 113-131.

Abstract. We consider azimuthal (or circular) *shear* of a hollow circular cylinder, composed of a homogeneous isotropic *compressible* nonlinearly elastic material. The inner surface of the tube is bonded to a rigid cylinder. The deformation may be achieved either by applying a uniformly distributed azimuthal shear traction on the outer surface together with zero radial traction (Problem 1) *or* by subjecting the outer surface to a prescribed angular displacement, with zero radial displacement (Problem 2). For an arbitrary compressible material, the cylinder will undergo both a radial and angular deformation. The class of materials for which *pure azimuthal shear* (i.e. a deformation with zero radial displacement) is possible is described. The corresponding angular displacement and stresses are determined explicitly.

4. Horgan, C. O., and Simmonds, J. G., "End effects in anisotropic and composite structures," *Proceedings of Army Symposium on Solid Mechanics,* Plymouth MA (1994), pp. 567-578, (ed. by F. D. Bartlett, Jr., S. C. Chou, K. Iyer and T. W. Wright).

5. Horgan, C. O., and Simmonds, J. G., "Saint-Venant end effects in composite structures," *Composites Engineering* **3** (1994), 279-286.

Abstracts: In these papers, we return to the issue of slow stress diffusion and Saint-Venant end effects in composite structures. Thin-walled structures such as aircraft and automotive parts, rocket casings, helicopter blades and containment vessels are often constructed of layers of anisotropic, filament or fiber-reinforced materials which must be designed to remain elastic. The extent to which *local* stresses, such as those produced by fasteners and at joints, can penetrate girders, beams, plates and shells must be understood by the designer. Thus a distinction must be made between *global* structural elements (where Strength of Materials or other approximate theories may be used) and *local* elements which require more detailed (and more costly) analyses based on exact elasticity. The neglect of end effects is usually justified by appeals to some form of Saint-Venant's principle and years of experience with *homogeneous isotropic elastic structures* has served to establish this standard procedure. Saint-Venant's principle also is the

fundamental basis for static mechanical tests of material properties. Thus property measurements are made in a suitable *gage section* where *uniform* stress and strain states are induced and local effects due to clamping of the specimen are neglected on invoking Saint-Venant's principle. Such traditional applications of Saint-Venant's principle require major modifications when strongly anisotropic and composite materials are of concern. For such materials, local stress effects persist over distances *far greater* than is typical for isotropic metals. In [4], we describe plane elastostatic problems where anisotropy induces such extended Saint-Venant end zones. The paper is a review and a comprehensive list of references is given to original work where details of the analyses may be found. A more comprehensive review, which included three-dimensional problems, effects of nonlinearity, and dynamic effects, is provided in [5]. The implications of such extended end zones due to anisotropy are far reaching in the proper analysis and design of structures using advanced composite materials (see Fig 1. attached).

6. Miller, K. L. and Horgan, C. O., "Conservation Properties for Plane Deformations of Isotropic and Anisotropic Linearly Elastic Strips," *Journal of Elasticity* **33** (1993), 311-318.

Abstract. Plane deformations of a rectangular strip, composed of an homogeneous fully anisotropic linearly elastic material, are considered. The strip is in equilibrium under the action of end loads, with the lateral sides traction-free. Two conservation properties for certain cross-sectional stress measures are established, generalizing previously known results for the isotropic case. It is noteworthy that in the first of these conservation laws only one of the off-axis elastic constants appears explicitly while in the second only the opposite off-axis constant appears explicitly. Such conservation properties are useful in assessing the influence of material anisotropy on Saint-Venant's principle, as well as in establishing convexity properties for cross-sectional stress measures. In particular, it is anticipated that the results should be useful in determining the extent of edge effects in the off-axis testing of anisotropic and composite materials.

7. Miller, K. L., "End Effects for Plane Deformations of an Elastic Anisotropic Semi-Infinite Strip," Ph.D. Dissertation, April 25, 1994.

Abstract. In the context of linear elasticity theory, Saint-Venant's Principle is often used to justify the neglect of edge effects when determining stresses in a body. This is valid in the case of an isotropic material. However for the more general anisotropic material, experimental results have shown that edge effects may persist much farther into the material than in the isotropic case and cannot be neglected. This research examines the effect of material anisotropy on the exponential decay rate for stresses in a semi-infinite elastic strip. A linear elastic semi-infinite strip in a state of plane stress/strain subject to a self-equilibrated end load is considered first for an orthotropic material and then for the most general anisotropic material. The problem

is governed by a fourth-order elliptic partial differential equation with constant coefficients. Conservation properties of the solution are derived to help in determining decay rate estimates. Energy methods are then used to provide lower bounds on the actual decay rate. Both analytic and numerical estimates are obtained in terms of the elastic constants of the material and results are shown for a set of specific materials. When compared with the exact decay rate computed numerically from the eigenvalues of a fourth-order ordinary differential equation, the results in some cases show a high degree of accuracy not achieved previously. Results of the type obtained here have several practical applications, for example, in the mechanical testing of anisotropic and composite materials and in assessing the influence of fasteners, joints, etc. in composite structures.

8. C. O. Horgan and K. L. Miller, "Anti-plane shear deformations for homogeneous and inhomogeneous anisotropic linearly elastic solids," *J. Applied Mechanics* **61** (1994) 23-29.

Abstract: Anti-plane shear deformations of a cylindrical body, with a single displacement field parallel to the generators of the cylinder and independent of the axial coordinate, are one of the simplest classes of deformations that solids can undergo. They may be viewed as complementary to the more familiar plane deformations. Anti-plane (or longitudinal) shear deformations have been the subject of considerable recent interest in *nonlinear* elasticity theory for homogeneous isotropic solids. In contrast, for the *linear* theory of *isotropic* elasticity, such deformations are usually not extensively discussed. The purpose of the paper [8] is to demonstrate that for *inhomogeneous anisotropic linearly elastic solids* the anti-plane shear problem *does* provide a particularly tractable and illuminating setting within which effects of anisotropy and inhomogeneity may be examined. We consider infinitesimal anti-plane shear deformations of an inhomogeneous anisotropic linearly elastic cylinder subject to prescribed surface tractions on its lateral boundary whose only nonzero component is axial and which does not vary in the axial direction. In the absence of body forces, *not all arbitrary anisotropic cylinders* will sustain an anti-plane shear deformation under such tractions. Necessary and sufficient conditions on the elastic moduli are obtained which *do allow* an anti-plane shear. The resulting boundary value problems governing the axial displacement are formulated. The most general elastic symmetry consistent with an anti-plane shear is described. There are at most 15 independent elastic coefficients associated with such a material. In general, there is a *normal* axial stress present, which can be written as a linear combination of the two dominant shear stresses. For a material with the cylindrical cross-section a plane of elastic symmetry (monoclinic, with 13 moduli) the normal stress is no longer present. For *homogeneous* materials, it is shown how the governing boundary-value problem can be transformed to an equivalent isotropic problem for a

transformed cross-sectional domain. Applications to the issue of assessing the influence of anisotropy and inhomogeneity on the decay of Saint-Venant end effects are described.

Presentations, DoD contact, Technology Transfer

1. "Effects of material anisotropy and inhomogeneity on cavitation for composite incompressible anisotropic nonlinearly elastic spheres" (with Debra A. Polignone). SIAM Conference on "Emerging Issues in Mathematics and Computation from the Materials Sciences," Pittsburgh, PA, April 1994.
2. "Effects of material anisotropy and inhomogeneity on cavitation for composite incompressible anisotropic nonlinearly elastic spheres" (with Debra A. Polignone). Invited lecture at Special Session on "Constitutive Formulations and Applications in Bioelasticity and Nonlinear Elasticity," 12th U.S. National Congress of Applied Mechanics, University of Washington, Seattle, June 1994.
3. "Anti-Plane Shear in Linear Anisotropic Elasticity" (C. O. Horgan). Invited lecture at Special Session on "Multiphase Elasticity and the Dundurs parameters," 12th U.S. National Congress of Applied Mechanics, University of Washington, Seattle, June 1994.
4. "A bifurcation approach to modeling void nucleation and growth in solids" (C. O. Horgan). Invited lecture at the Flight Dynamics Laboratory, Wright Patterson Air Force Base, Ohio, August 1994.
5. "On axisymmetric solutions in compressible nonlinearly elastic solids" (C. O. Horgan). Invited lecture at Special Session on "Elasticity," 31st Annual Meeting, Society of Engineering Science, Texas A & M University, October 1994.
6. Research visit to Boeing Aircraft Company, Seattle, WA, July 1st 1994 (C. O. Horgan)
7. Several colloquium lectures delivered at various universities.
8. Results of research widely distributed to NSF, ARO, AFOSR, NIST, and private industry.
9. Continuous contact maintained with Dr. A. Nachman, Program Director, Applied Analysis, AFOSR via telephone, preprint and reprint submission, AFOSR Annual Data.

Awards.

1) C. O. Horgan began term of office as a member of the Board of Directors, Society of Engineering Science, Inc., January 1994.

2) C. O. Horgan was named the Wills Johnson Professor of Applied Mathematics & Mechanics, University of Virginia, July 1, 1994.

Edge Effects in Composite Structures

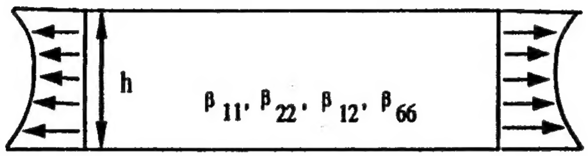
C. O. Horgan and J. G. Simmonds
School of Engineering & Applied Science
University of Virginia

Assumptions

I. Orthotropic material

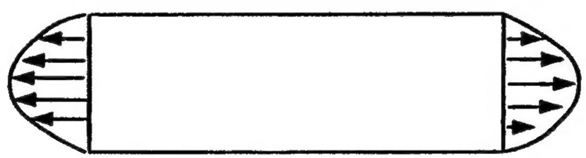
- 2-D plane stress/strain
- Linear elasticity

(extension problem)



II. Statically equivalent loading

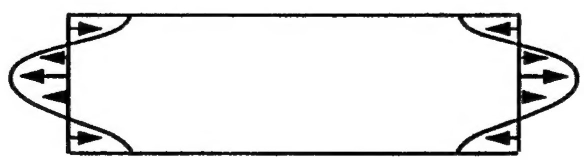
(same resultant force and moment)



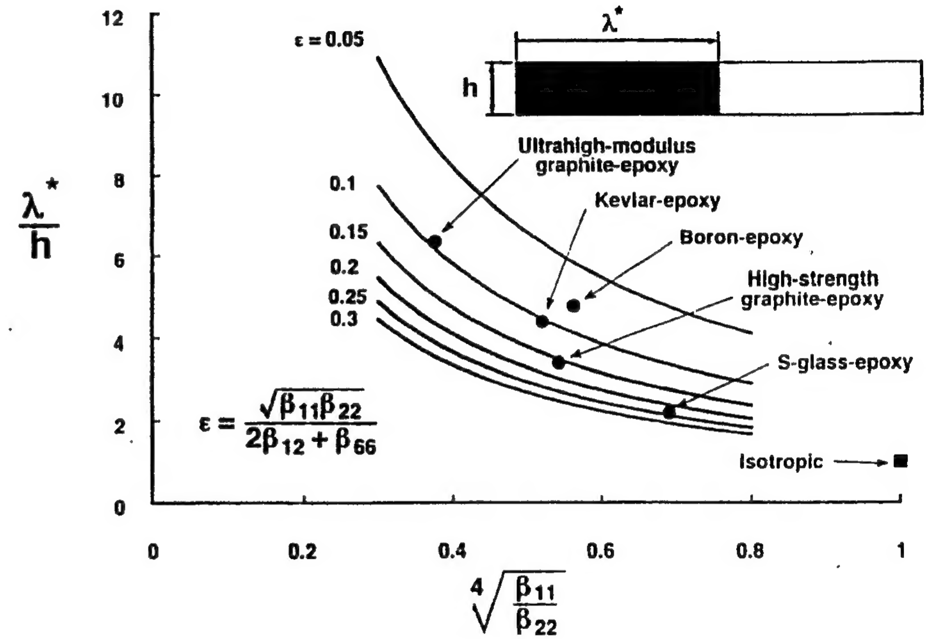
III. Self-equilibrated end loads

- The stress fields decay from the ends
- Rate of decay can be found
- Characteristic decay lengths

(Superposition of I and II)



Results



Decay length λ^* for semi-infinite orthotropic strips subjected to self-equilibrated end loads.

A comparison between several commonly used composite materials is illustrated for specially orthotropic materials. The characteristic decay length (i.e., the distance over which end effects decay to 1% of their end values) versus a nondimensional material parameter is plotted and the results for various materials are indicated by the dots shown on the curves. The decay length for an isotropic material is shown by the dark square. It is seen that the latter has the *smallest* decay length and that this is approximately equal to the width of the strip. This figure can be used directly in the design process to account for anisotropic end effects.

A Technology Transfer Example

A Boeing/NASA Advanced Technology Composite Aircraft Structures (ATCAS) Program has been active since 1989. The primary objective of this program is to:

> "Develop an integrated technology (manufacturing & structures) and demonstrate a confidence level that permits cost-and weight-effective use of advanced composite materials in primary structures of aircraft with the emphasis on pressurized fuselages."

In this program, a section of a widebody aircraft (244" dia) just aft of the wing/body intersection is being analyzed by the Boeing Commercial Airplane Group in Seattle, Washington. Sandwich structures are being used for the side and keel of this section. Compression testing of laminate coupons indicate the need to incorporate Saint-Venant end effects in interpretation of the test data. The work of the PI's is being utilized in this effort. One of the P.I's (C. O. H.) visited the Boeing Group in Seattle on July 1, 1994 to consolidate this interaction. It is planned to engage in collaborative research with the Boeing scientists (Dr. W. A. Avery, coordinator). One objective is to develop a systematic testing program to be carried out by Integrated Technologies, Inc. (Intec), Bothell, WA, under subcontract to Boeing. Preliminary tests by Intec have indicated problems due to end effects in the sandwich panels under investigation. It is anticipated that the theoretical results obtained in our research program will have direct application to these problems. In fact, the interaction with the Boeing/Intec group is providing additional motivation and stimulus to our efforts in understanding the extent of Saint-Venant end effects in advanced composite materials and structures.

The Boeing Company
P.O. Box 3707
Seattle, WA 98124-2207

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Professor Cornelius Horgan
Department of Applied Mathematics
University of Virginia
Thornton Hall
Charlottesville, VA 22903-2442

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Dear Professor Horgan,

I am writing to you in response to your letter of November 12, 1993 to Scott Finn. Thank you for sending copies of your papers on St. Venant effects. Mike Nemeth is correct in his perception that Boeing is interested in St. Venant effects in composite structures. In fact, I independently searched the literature for papers on St. Venant effects and came up with many authored by you and your colleagues. I would like to take this opportunity to give you a brief background on our project and then describe some of the technical issues that I think might interest you.

Boeing is currently funded under NASA's Advanced Composites Technology (ACT) program to develop the materials, structures, and manufacturing technology necessary to build a fuselage section for a widebody commercial transport. Boeing calls its program the Advanced Technology Composite Aircraft Structure (ATCAS) program. In this program we have chosen for study a section of a widebody aircraft (244" dia.) just aft of the wing/body intersection. The section is approximately 32 feet long. This section is chosen because its geometry presents most of the technical challenges of producing a composite fuselage structure. It contains landing gear cutouts, cargo door cutouts, window cutouts, and complex loading due to its location with respect to the wing. The program started in May of 1989 and Phase B is scheduled to conclude in 1995. During these 2 phases we have designed, built and tested several subcomponent composite structures. Most of the early work concentrated on the fuselage crown, which is a skin/stringer design with relatively thin skins. More recently, we have moved on to the keel and side structure. These structure are much more difficult to design due to the presence of a complex load state and the number of cutouts. In 1995 Phase C is scheduled to begin. During Phase C Boeing will be contracted to build a full barrel Section 46.

Although we chose skin/stringer as the structure type for the crown, we have chosen sandwich structure for the side and keel. We are using Hercules' AS4/8552 for the skin and Hexcel's HRP honeycomb core. In the past few months we have been conducting compression tests of sandwich structure with impact damage, notches, and undamaged. All of the specimens have been instrumented with strain gages and on some we have collected photoelastic data.

As you are probably aware it is common practice to use relatively short coupons for compression tests in order to prevent global buckling. Boeing's compression-after-impact (CAI) coupon is 6" x 4", which presents an aspect ratio L/W which is only 1.5. During our program we conducted a compression test of a sandwich specimen with a 1" hole. The dimensions of that coupon were 18" x 12". We did not observe the hole size strength effect we expected and came to the conclusion that the load wasn't evenly introduced into the specimen; hence, the hole never saw the expected stress concentration. In another experiment we performed compression testing of 5" x 7" and 5" x 12" open hole (1") solid laminate coupons. Photoelastic data was collected for both specimens. The results showed that better load introduction was obtained in the longer specimen. These seem to point to St. Venant effects and/or specimen fixturing conditions as the reasons for uneven load introduction. In any event, it appears that L/W ratios on the order of 1.5 are insufficient for credible test data. We have somewhat arbitrarily tried to use L/W rations of at least 2 on this program.

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I would be most interested in having some informal discussions with you on these issues when you visit Seattle next year. Perhaps you can give us some insight on St. Venant effects in sandwich composite structure, and maybe you might leave with some ideas for future research. I'm looking forward to meeting you.

Sincerely,

William B. Avery, Ph.D
M/S 6H-PJ
206-234-0444